Chapter 2: Literature Review

2.1. Introduction

In this chapter a review of current literature related to the field of study is presented to justify the choice of topic for the research and how the methodology adopted is relevant for addressing the proposed research questions. The study has been influenced by research emerging from mainly three perspectives and this chapter will focus on these perspectives in separate sections. Firstly, in Sections 2.2 and 2.3 literature related to mathematics learning in the vocational education context and students’ attitude towards mathematics is presented to highlight the context and relevance of current study. Secondly, in Section 2.4 a review of literature on theoretical underpinnings concerned with the application of new learning technologies in the teaching and learning of mathematics is presented to show their relevance for this study. Furthermore, in this section literature related to learning theories relevant to the design and teaching of mathematics with the help of new learning technologies is also presented to support and justify the methodology used in this study. Finally, the last section of this chapter, Section 2.5, is devoted to the issue of designing online learning environments for mathematics. This section focuses particularly on literature related to web-based learning in mathematics classrooms.

2.2. Mathematics in Vocational Education

2.2.1 Mathematics in Society

Mathematics occupies an important place in society and teaching and learning of mathematics has been an important part of school since ancient times (AAMT, 1996). Most ancient civilizations such as Hindu, Greek and Roman included elementary mathematics in their education system but it remained largely confined to male children of upper class and wealthy citizens (O'Connor & Robertson, 2000). In those days mathematical knowledge was considered a lesser intellectual pursuit than metaphysical, moral philosophy and religion because it was associated with practical and utilitarian ideas of everyday trade and calculations related to worldly affairs. For
example, classical education of medieval Europe included teaching of mathematical fields of arithmetic and Euclidean geometry and apprentices of trades such as masons, money-lenders and merchants were expected to learn this practically oriented mathematics.

In contemporary society mathematics is considered a backbone for science and technology developments and mathematics has become an integral part of school curriculum. Mathematics content and teaching methods are constantly debated and governments take a keen interest in making sure that the quality of mathematics teaching is maintained in State funded schools. In recent years many technologically and economically advanced countries have begun to monitor comparative performance of school age children in mathematics and science through surveys such as Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMMS). Results from these comparative studies are reported widely in media and influence educational policy developments in concerned nations (Bosse, 2007).

Worldwide educational reforms have pursued development of standards for teaching and learning of mathematics. For example in the United States the National Council for Teaching of Mathematics (NCTM) developed new standards for teaching of mathematics in schools in 1989 to meet the needs of students entering a more technologically advanced and complex workforce (Crosswhite, Dossey, & Frye, 1989). The NCTM standards were further refined in 2000 with the publication of NCTM Principles and Standards for School Mathematics document (Bosse, 2007). The NCTM principles recognise that “technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning” (National Council of Teachers of Mathematics, 2000, p. 11).

In the United Kingdom Professor Adrian Smith’s (2004) inquiry into post-14 mathematics education grew out of the concern that current curriculum, assessment and qualifications frameworks are failing to meet the needs of many learners and do not satisfy the requirements and expectations of employers and higher education institutions. One consequence of this report has been the establishment of a National Centre for Excellence in the Teaching of Mathematics (NCETM) in 2006, which aims
to enhance the professional development for mathematics teachers in all education sectors in England.

In Australia the Statements of Learning for Mathematics (Curriculum Corporation, 2006) has been developed with a collaborative effort from States and Territories with an objective of serving as a nationally consistent framework for the development of new curriculum and reforming mathematics education. The statements highlight the dynamic nature of mathematics and note that:

Mathematics knowledge has been developed across cultures throughout history and continues to develop today. Mathematics education responds to social change, developments in mathematics, new technologies and new approaches to mathematics enquiry. (p.2)

The changing nature of mathematics and the need for mathematics curriculum to respond to these changing needs was also emphasised in the previous national statement on mathematics (Australian Education Council, 1990) which noted that the rapid growth of mathematics and new technologies were influencing the way mathematics is produced and applied and that access and success in mathematics is necessary for the economic well being of Australia. The statement predicted that demand for mathematically skilled people in Australia will rise but supply fall. The document noted that due to a shortage of skilled workers worldwide, Australia can no longer “expect immigration to fulfill our shortfall” (p. 15). Indeed, these predictions about the skills shortages in Australia have become a reality (Khoo, McDonald, Voigt-Graf, & Hugo, 2007). Another important aspect of the Australian Education Council’s National Statement on Mathematics was its clear endorsement of the use of technology in teaching mathematics. It pointed out that:

There have been considerable advances in computing software suitable for use in school mathematics. There is a range of ways in which computers can be used in mathematics classrooms including 'number crunching', data analysis, as a simulation device, graphics, symbol manipulation and spreadsheets. Each of these uses has implications for what is most usefully taught in mathematics and how it is taught. (p.31)

The ideas espoused by these national statements on mathematics have been central in guiding curriculum frameworks in mathematics such as the Victorian Essential
Learning Standards (VELS) (VCAA, 2008). The VELS on mathematics clearly outlines that school mathematics needs students to be able to demonstrate useful mathematical and numeracy skills for successful general employment and functioning in society and solve practical problems with mathematics, especially industry and work-based problems (p.4).

Concerned with the state of numeracy and mathematics learning in schools the Queensland government has produced a Framework for Action (Education Queensland, 2007) to support teachers of mathematics in offering high quality mathematics and numeracy programs. The Framework for Action aims at helping students develop capabilities to lead numerate lives and meet the numeracy demands of all learning areas of the curriculum. In this framework also, the ability to use technology in teaching and learning of mathematics is an important aspect of teacher’s knowledge and pedagogy (p. 8).

The Australian Association of Mathematics Teachers (AAMT) have adopted Standards for Excellence in Teaching Mathematics in Australian Schools in which it acknowledges that excellent teachers of mathematics have a strong knowledge base to draw on in all aspects of their professional work and that they are aware of a range of effective strategies and techniques for utilizing information and communication technologies in teaching of mathematics (Australian Association of Mathematics Teachers, 2006).

Although there is widespread recognition of the importance of mathematics in today’s highly technological society research has also found that there is a highly visible trend of diminishing enrolments in mathematics courses in higher education (Forgasz, 2006a). Furthermore, although in secondary schooling there is an increased participation in learning and more students are staying on in schools to finish their senior secondary schooling, in terms of mathematics they appear to be opting for the easier mathematics option. One consequence of this trend appears to be that students enrolling in vocational education programs in TAFE courses often do not possess pre-requisite knowledge and skills in mathematics. There is a well recognised need in the sector to provide educational intervention and support to enable these students to succeed (Wilson, 2007).
In the context of post secondary vocational or technical education the American Mathematics Association of Two-Year Colleges (AMATYC) appears as a leading forum devoted to the issues of mathematics teaching and learning (FitzSimons, 2002a). The Organisation has more than 100 institutional members from the U.S. and Canada and has been committed to providing a national forum for the improvement of mathematics instruction in the first two years of the college.

The AMATYC has developed sets of standards for intellectual development, content and pedagogy to provide a foundation for mathematics teaching in the two-year college programs. The intellectual development standards include: problem solving, modelling, reasoning, connecting with other disciplines, communicating, using technology, developing mathematical power and linking multiple representations (AMATYC, 2005). The AMAYTC standards for pedagogy outline guidelines for instructional strategies in active student learning and include: teaching with technology, active learning, making connections, using multiple approaches, experiencing mathematics and interactive lecturing (AMATYC, 2005). The standards developed by AMATYC emphasise that technology should be an essential feature of mathematics teaching and learning and note that:

*Technology continues to change the face of mathematics and affect the relative importance of various concepts and topics of the discipline. Advancements in technology have not only changed how faculty teach, but have also changed what is taught and when it is taught. Thee many types of technologies with varied uses can deepen student learning and prepare students for the workplace. (p.11)*

In the vocational education sector in Australia, however, no such standards or national statements regarding the teaching and learning of mathematics can be located. In Australia, mathematic teaching in the vocational education field remains a marginalised and isolated activity and decisions about curricula rest with industry training boards (FitzSimons, 2002a). Evidence from the field suggests that experienced VET practitioners often draw upon their own resources to adapt existing
curricula in mathematics to create innovative and meaningful ways of improving teaching and learning of mathematics.

### 2.2.2 Workplace mathematics

There is a growing gap between what is taught as mathematics in schools and what mathematics is needed at workplaces. Research from workplaces has shown that mathematics used in workplaces is highly context dependent and situated in the practices of the workplace (FitzSimons & Mlcek, 2004; Marr, 2007; Zevenbergen & Zevenbergen, 2004). Increasing use of technology has also made a significant impact on mathematics practices of workplaces. Research is showing that technomathematical literacy is needed in technology rich workplaces of today (Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002; Noss, Hoyles, Bakker, & Kent, 2005). These findings have direct bearing on vocational education programs where mathematics is taught to prepare the workforce for the future. If we want the gap between what is taught as mathematics in formal vocational training and what knowledge and skills of mathematics are needed at workplaces then we need to adapt our mathematics content and teaching approach to be more relevant to the current and future workplace.

Zevenbergen (2005) in a longitudinal study comparing numeracy practices of younger generation shop assistants (people aged 22 or less referred to as Millennials by author) with non-millennial employers and job placement workers noted that young participants were predisposed to use technology and were seen to “defer the cognitive labour to technology i.e. cash registers, computers, or calculators” (p.7). She points out that the young workers predisposition to use technology manifested in their willingness to use estimation and ability to problem solve more effectively than their employers and older peers. These findings have implications for training providers that continue to follow a traditional format and view the use of calculators as a form of “laziness and incompetence” (p.9).

In relation to the mathematical needs of workers in a modern technological society there is a view that suggests that as new technologies make mathematics invisible
most workers would not need to know much about mathematics. However studies of mathematical use in workplaces has shown that technology use is increasing and workers need to demonstrate different mathematical skills than traditionally taught in school mathematics. Instead of having the fluency in performing explicit mathematical procedures using pen and paper workers are now required to show fluency with using and interpreting output from an IT system and software. They are also encouraged to demonstrate an understanding of the mathematical models deployed within the IT system in order to inform workplace judgements and decision-making (Bakker, Hoyles, Kent, & Noss, 2006).

In a research study focussing on how to enhance techno mathematical literacy of workers at a finance enterprise selling mortgages, Bakker et. al. (2005) observed that the mortgage provider despite acknowledging that sales employees were narrowly trained for future needs, pointed out that the company deliberately avoided training in explicit mathematical ideas underlying IT based financial calculations in order to avoid alienating most of their sales employees. In training sessions to enhance techno-mathematical literacy of sales employees the researchers adapted the idea of using customised spreadsheets to open up “black box” calculations for investigation because the process allowed for manipulation and exploration of variables (p. 6). The researchers noted that this way of learning kept algebraic formalism “encoded” within the spreadsheet structure in such a way that the learner could see rather clearly what the formalism was doing (p.9). The focus of training in techno mathematical literacy is to guide “learners to use mathematical ideas appropriately, rather than guiding them to do the explicit mathematical calculations involved” (p.9).

In today’s highly technological society most financial services institutions are increasingly using online tools to help customers choose from a range of products and services. Many of these tools are based on sophisticated mathematical calculations taking place in the “black box”. Students currently doing vocational education courses in preparation for working in the finance sector companies will be required to use these online tools and sophisticated calculators in their everyday workplace practice. It therefore becomes imperative for vocational education programs to prepare students to meet the challenges they will face in technology rich workplaces of the future.
Workplace mathematics skills are context dependent and highly specific to particular contexts and it is not possible for formal training to include each situation and context. In this situation it also becomes critical that formal training consciously aim to help learners develop meta-cognitive strategies such as learning to learn, critical thinking, planning and problem solving (FitzSimons & Mlcek, 2004). Furthermore, mathematical knowledge and skills developed in school and vocational education play an important underpinning role in workplace numeracy practice. FitzSimons, Mlcek, Hull and Wright (2005) argue that workplace numeracy education cannot be approached from a traditional 'school mathematics' outlook (p.9). In their workplace study these authors noted that numeracy in the workplace involves the practical application of rational numbers and the metric measurement system with contextualised approximations and estimations in critical calculations, often with other workers. They point out that workplace mathematics use differs markedly from the traditional conception of mathematics education as an “abstract, rule-bound, individual activity, with one correct answer (usually a number, an algebraic expression, or a standard graph), and where mistakes are temporary setbacks” (p.6).

A number of studies focussing on mathematics learning and use from Australian workplaces confirm the view that mathematics used in industry is intertwined with technical expertise at all occupational levels. Research shows that employees at workplaces have more control over how they solve their problems compared with traditional classroom mathematics and the mathematical correctness or precision required for completing a task at workplaces depends on constraints such as time and money (Buckingham, 2001; FitzSimons, 2003; Marr, 2007; Zevenbergen, 2005).

This emerging understanding of how mathematics is used and applied at modern workplaces has immediate relevance for the design of learning in vocational education. Learners in vocational education and training need to be presented with opportunities to become familiar with and use artefacts from relevant workplaces. At the same time it needs to be recognised that every workplace context can not be incorporated into formal training because enterprises have become highly specialised. Therefore there is a need to include meta-cognitive strategies such as problem solving and critical thinking along with mathematical knowledge and skills (FitzSimons, 2003).
During this study, the development of a web based learning environment to support learning of mathematics for vocational students aimed at enhancing technomathematical literacy of students by providing them the opportunity to use the online tools and calculators from real world contexts in classroom based mathematics learning. During teaching online learning activities were mixed with face to face mathematics instruction to promote the use of technology in solving mathematics problems and helping students develop meta-cognitive skills of critical thinking and problem solving.

2.2.3 Mathematics and Vocational Students

In recent times there has been an increased focus on the VET sector of education internationally. This is mainly in response to global economic change and government and industry now view VET as a major factor in the drive to be internationally competitive. Keating, Medrich, Volkoff and Perry (2002) point out that the VET sector provides great flexibility in course length, content, location and modes of delivery and it is seen as providing both industry-specific skills and more generic workplace skills. In response to a shortage of skilled workers to meet the needs of industry there is a renewed emphasis on invigorating technical and vocational education programs via increased funding for VET education, additional pre-apprenticeship places and opening new technical training centres (Department of Education and Training, 2006).

Interestingly while there is a renewed interest and a move to expand vocational education and training provision there is also a general trend of a declining number of students studying subjects like advanced mathematics, physics and chemistry in their senior years of secondary schooling (Department of Education Science and Training, 2003; Forgasz, 2006a). There is also a dearth of qualified mathematics teachers in the school sector and many students learn mathematics from teachers who do not have mathematics qualifications. Thomson and Fleming (2004) in their analysis of TIMSS data note that 30% of teachers teaching mathematics to year 8 students in Australian schools do not have either education mathematics or mathematics as their major area
of study. It is cause for concern that teaching that does too little to stimulate curiosity, problem solving and depth of learning among school students would not encourage them to continue with mathematics in their senior years of schooling and beyond (Department of Education Science and Training, 2003).

Historically students opting for vocational education programs in TAFE colleges are not the highest academic achievers. These students enrol in vocational education programs largely because they may have an aptitude for a practically oriented occupation or they may do a TAFE course as a pathway to higher education degree. However, Anderson (2005) has refuted the assumption that VET students enrol in courses mainly for extrinsic and work-related reasons. He notes that aspirations for personal growth and development and career change are common motivators for students’ course choices in the VET sector particularly for students in the age group of 25 and above (p.12). In an Australian survey of VET students Anderson (2005) found that student's transition into VET courses appears to be non-linear and diverse. He found that in a significant number of cases they do not come to vocational education directly from secondary schooling. In this study the respondents came to VET from several other forms of education and training including school (17%), transitioning out of unemployment (10%), out of employment to education and training (42%), or into study from home based work (12%) (Anderson, 2005). The survey also shows that individuals opting for VET pathways typically follow zigzag trajectories, frequent interruptions or changes in direction. They use VET qualifications to navigate changing “career trajectories” during their working lives (p 9).

Young vocational education students also have to make a transition from school based learning practices to adult learning practices of vocational education where they are expected to be more self-directed, autonomous and responsible for decision making (Choy & Delahaye, 2002). In the vocational education context the term andragogy is used which usually refers to learner-centred teaching practices as pioneered by Knowles (1980). Knowles’ assumptions regarding andragogy are based on the disposition of adult learners and Choy and Delahaye (2002) articulate these as:
- The need to know: adults like to connect learning with their lives readily and prefer active learning which fits into the context of their life activities;
- The learner’s self-concept: adults see themselves as individuals who have the capacity to make decisions for themselves;
- The role of learners’ experience: adults possess life experiences which serve to express their self identity and form valuable learning resources;
- Readiness to learn: adults show a self-realisation of the need to learn;
- Orientation to learning: adults are motivated to learn because they understand the value of learning in enhancing their ability to address issues of their daily lives;
- Motivation: adults are generally intrinsically motivated to learn.

Most of these andragogy principles are reflected in VET sector learning and learners are expected to take responsibility for their learning in self-paced learning materials. Indeed, competencies listed in particular training packages are realistic and located in the context of real workplaces.

In a study aimed at finding out if young people age 17-24 years enrolled in a VET course have an orientation to learning that benefits from andragogy practices, Choy & Delahaye (2002) surveyed 266 young people enrolled in metropolitan TAFE colleges. They noted that young people preferred mainly the social aspects of andragogy but were less willing to accept responsibility for learning associated with adult learners. They were also more inclined to view teachers as experts and transmitters of knowledge. These findings have clear implications for teachers involved with the learning of young adults at TAFE courses. Young adults seem to carry with them practices they are familiar with from their school years and simply expecting them to be able to take responsibility of their learning in VET through self paced and flexible learning formats is unlikely to be successful (Keith & Javed, 2004). In the research by Choy and Delahaye (2002) although young students indicated that they wanted to be treated like adults and showed a preference for a closer relationship with teachers, for structured course work and organised assessment procedures, their view of the teacher as an expert and responsible for directing their learning has important implications for this research. As mathematics teachers responsible for designing learning opportunities for young students we needed to find a balance between the andragogy
principles that allow us to construct learning activities that offer relevance and opportunity for self-paced learning on the one hand and pedagogical needs of learners who expect teacher-directed structuring of learning activities on the other.

The pattern of work and education also bears significant influence over participation and success of students in vocational education programs. It is no longer the case that students first undertake full time study and after completion of this study enter into full time employment. Current students hold part-time and casual jobs while undertaking full time or part-time post school studies in VET or higher education. In a national online survey of VET students enrolled at Registered Training Organizations in Australia, Anderson (2005) found that 67% of those enrolled in a full-time VET course were engaged in casual employment whereas 74% of those enrolled in part-time VET course were engaged in full-time employment. This trend of concurrent participation in work and education appears to be closely associated with the frequency of career change among individuals undertaking VET courses. Gabb, Milne and Cao (2006) point out that the personal choices made by students in relation to work and education are highly relevant and put pressure on educational institutions for more flexible delivery of educational products.

Another common assumption about vocational education students currently prevailing is that the young generation of today is totally immersed in technology. It is argued that today’s generation of learners, sometimes referred to as generation Y or Net generation (2005), has grown up with all the new technology to the extent that it is second nature to them. A second assumption accompanying this argument is that generation Y will be totally at ease with learning with new technologies and show a readiness to learn with these new technologies.

In a review of research on learner readiness to adopt online learning and flexible learning in the VET sector Robertson (2007) found that it can not be assumed that learners will automatically perceive that a technology they regularly use for non-educational purposes will be useful to support their learning. Review of research suggests that the popularity of technology on its own is insufficient to guarantee its transfer into educational use by generation Y learners. This observation is important for teachers in VET programs because, although young students may appear to be
totally at ease with using computers and Internet technologies, to engage them in learning via new technologies needs careful planning when designing technology that will support their learning.

Research into the use of technology in mathematical tasks by children of the baby boomer generation (and younger) has found that they prefer to defer the cognitive labour of calculations and other mathematical thinking to technology. In a case study research by Zevenbergen and Zevenbergen (2004) found that the need to calculate accurately by the young generation seems to be deferred to technological tools – computers, calculators, and other industry specific equipment. The skills of estimation are central to their work as is having an intuitive feel for a situation. The researchers note that the old skills of accuracy, meticulous mental calculations and measurement have been displaced by skills of estimation, problem solving, and use of technology. These findings have implications for practices of teaching vocational mathematics where holistic thinking, problem solving, estimation, technology, and intuitive thinking will need to be included in curriculum planning and design.

2.3. Attitude towards mathematics

Many students in vocational education, especially adult learners returning to study after a gap of some years, tend to show a level of mathematics anxiety usually arising due to their negative experiences of learning mathematics at school (FitzSimons, 1998). Students’ perceptions that they can not understand mathematics and that it is a hard subject is linked to an image students have of mathematics as an abstract collection of rules and processes (Kaput, 1995). Anxiety in mathematics generally manifests itself in the form of feeling of tension, apprehension or fear (Ashcraft, 2002). This anxiety often leads to low self-confidence in doing mathematics and impacts on mathematical performance of learners. Confidence of learners in their mathematical abilities and their attitudes and beliefs towards mathematics affects learning of mathematics (Coben, 2003; FitzSimons, 2002b).

The literature on attitude towards mathematics draws on a range of theories and definition of attitude. Aiken (2000) defines attitude as a “learned predisposition to
respond positively or negatively to a specific object, situation, institution or person” (p.248). Attitudes are distinguished from beliefs in that attitudes are moderate in duration, intensity and stability and have an emotional content, while beliefs become stable and are not easily changed (McLeod, 1992).

Ma and Kishore (1997) conceptualise students’ attitude towards mathematics as “an aggregated measure of a liking or disliking of mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or useless” (p. 27).

Lefton (1997) views attitudes as long lasting patterns of feelings and beliefs about other people, ideas or objects that are based in a person’s past experiences and shape their future behaviour. Lefton (1997) identifies three dimensions of attitudes as cognitive, emotional and behavioural and points out that each dimension serves a specific function. The cognitive dimension of an attitude consists of thoughts and beliefs. The emotional dimension involves evaluative feelings such as like and dislike. The behavioural dimension of an attitude determines how people actually show their beliefs and evaluative feelings. In this study I was aware that vocational education students exhibit a wide range of attitude towards mathematics based on their educational experiences and personal circumstances but my interest was in establishing how their enjoyment (emotional attitude) and their perception of value (cognitive dimension) was influenced by the implementation of an online learning environment to support classroom based learning.

The research literature on measurement of attitude towards mathematics provides an extensive range of measuring scales developed over the last thirty years. Fennema and Sherman (1976) attitude scales have been one of the most popular tools for measuring mathematics attitude of school age children. This scale has 108 items and takes about 45 minutes to administer (Barkatsas, 2005; Tapia & Marsh, 2004). The Fennema-Sherman Mathematics Attitude Scale is made up of nine subscales including Attitude towards Success in Mathematics Scale, Mathematics as a Male Domain Scale, Father/Mother Scales, Teacher Scale, Confidence in Learning Mathematics Scale, Mathematics Anxiety Scale, Effactance Motivation Scale in Mathematics and Mathematics Usefulness Scale. Although this scale has been used extensively a
number of researchers have questioned its validity and the integrity of scores generated by these scales (Tapia & Marsh, 2004). As this scale uses statements appropriate for school aged children it was not considered appropriate for use with the students who were young adults in this study.

Another popular attitude scale designed to measure the attitudes toward mathematics is the Mathematics Attitude Scale (Aiken, 1974). This scale measures general attitude towards mathematics and comprises of two subscales of 10 items each covering students enjoyment of mathematics with items such as “mathematics is enjoyable and stimulating to me”, and their perceptions of its value with inclusion of items such as “mathematics is a very worthwhile and necessary subject”. The 20 items in this scale comprise half positive statements and half negative statements. The original scale includes Likert scale format but a revised version used by Drexel University (2001) uses a Yes/No format for response. The Aiken Mathematics Attitude Scale is reported to have a test/retest reliability of .94 (Bassette, 2004).

Taylor (1997) suggests that the relative simplicity and brevity of Aiken’s Mathematics Attitude Scale provides significant advantages to educators and researchers in the field. The scale has been popular with researchers in the post secondary and adult and community education. Recent studies by Chapman (2003), Bassette (2004) and Yushua (2006) have used this scale with pre-tertiary and vocational education students. I chose this scale for my study due to its relevance for tertiary level students and its simplicity and brevity for ease of administration and analysis.

Bassette (2004) used Aiken Attitude Scale to measure the attitude of mature age students enrolled in a developmental basic arithmetic course at Prince George's Community College in Maryland. A sample of 329 students enrolled in day and evening classes participated in the study. Students were given the Aiken Mathematics Attitude Test at the start of the semester as pretest and again at the end of the semester as posttest. The study aimed to assess the difference of the initial and exiting attitudes towards mathematics and academic outcomes of students. The study also aimed at finding out if there were any association between initial placement test scores and students pretest attitude scores and final exam scores. Demographic variables such as
age, gender and ethnicity were also used as criteria when comparing students’ attitudes and performance in placement and final examination.

Results on the Aiken attitude pretest showed that 61% of students score 40 or less on a scale with maximum 80 points showing a low or negative attitude towards mathematics. Only 39% students scored more than 40 showing a positive attitude towards mathematics in pretest. Overall comparison of pretest and posttest attitude scores showed a moderate association (p=.41) and pretest attitude scores of women were positively related to the placement score and the final examination score. The study found no significant difference in attitude between students who passed and students who failed. The study also reported that students aged 50 and above scored better marks in final exam than other groups. In contrast students aged 21 and less scored less than other groups in the final examination. Overall Bassette’s (2004) study shows only a weak association between attitude and performance in mathematics.

In a study of effects of blended e-learning on mathematics and computer attitudes in a pre-calculus algebra course in a pre university program in a Saudi Arabian university Yushau (2006) used Aiken Mathematics Attitude Scale to compare students attitude at pre and post course stages. Along with mathematics attitude the study also measured computer attitude at pre-test and post-test stages. Seventy students from a prep year program participated in this experiment where three face-to-face teaching sessions per week were complemented with an online session where MATLAB and other online resources to support mathematics were available to students. Using a WebCT platform course material, solutions to past exam papers and quizzes were available for students. Some problems were posted on the course home page on a weekly basis for students to solve and submit online. In addition all course related announcements were also made using the online home page. Results from the study compared pre and post attitude towards mathematics and also towards computers.

The study reported no significant difference in students’ attitude towards mathematics and the mean scores at both pre and posttests suggest that students maintained a mainly positive attitude. However there was a slight decrease in the mean score for mathematics attitude at the posttest stage. On the computer attitude scale also there was no significant difference but surprisingly on two subscales- computer confidence
and anxiety, students post scores were significantly lower than the pretest level. Yushau (2006) suggests that because the university places more rigorous and higher standards than students expect compared to their high school studies they felt overworked compared to the other group who were taking the normal lecture in the traditional mode only. The study indicates that blended learning adds new dimensions to learning and students’ preparedness to undertake online-based learning may play an important role in successful implementation. Also, traditional assessment modes and quality standards can make technology appear to be an extra burden rather than a means for exploration, learning and concept development.

In a pre-tertiary foundation course in Introduction to Quantitative Methods Klinger (2005) studied the profile of attitudes, anxiety and self-efficacy beliefs in mathematics using a pre-post survey with 160 students at an Australian tertiary institute. The study developed a 95-point survey by adapting items from three different attitude, anxiety and self-efficacy scales. Klinger found significant improvement in students’ perceptions and beliefs of mathematics after nine weeks of teaching but noted that if students’ tertiary education goals are blocked or their progress impeded because of their perceptions of mathematics it becomes necessary to first challenge their negative attitude and perceptions towards mathematics. It was noted that attitudinal negatives towards mathematics could fuel avoidance behaviour in students resulting in a diminished self-efficacy belief, heightened anxiety leading into withdrawal from mathematics learning. In vocational and further education courses also a similar trend is observed (Callan, 2005; Coben, 2003; Klinger, 2005) and one of the purposes of my study was to create a learning environment that creates opportunities for changing students’ negative attitude towards mathematics.

In recent years a renewed focus on the significance of affective factors in mathematics learning has been highlighted by the emergence of a range of new instruments and scales for measuring attitudes towards mathematics. Tapia and Marsh (2004) have developed an Attitude Towards Mathematics Inventory (ATMI), which consists of a 49-item survey and includes self-confidence, value, enjoyment and motivation factors. The ATMI is designed particularly for secondary school students and items are suited to the context of school-aged students. With increasing application of technology in
teaching and learning of mathematics new scales for monitoring students’ attitude to learning mathematics with technology have also emerged.

Barkatsas (2005) developed a Mathematics and Technology Attitude Scale (MTAS) for middle secondary years students with the intention of including mathematics confidence, confidence with technology, attitude to learning mathematics with technology, affective engagement and behavioural engagement as five affective variables relevant to learning mathematics with technology. This 20-item scale consisted of five subscales with 4 items each belonging to five subscales. Trials of MTAS scale with 350 students from year 8 to year 10 classes from Victorian schools in Australia produced results that suggests strong or acceptable degree of internal consistency in each subscale. The scale is simple and brief and takes only about 10 minutes to administer but its usefulness is confined to cohorts in the secondary schools due to the specific references to graphic calculators and school context.

In the area of technology enriched mathematics learning at the undergraduate level a number of Australian researchers have been active in designing and testing affective measure scales and instruments. Fogarty, Cretchley, Harman, Ellerton and Konki (2001) developed an Attitude to Technology in Mathematics Learning Questionnaire to measure three affective factors – mathematics confidence, computer confidence and attitude to technology in learning mathematics. The Likert type measurement scale consisted of 34 items and focused on identifying and measuring attitudinal factors that mediated the effective use of technology in learning mathematics. In a study to validate the questionnaire it was trialled on a cohort of 289 students undertaking algebra and calculus course as part of their undergraduate studies in a (Education Queensland) university. Cronbach’s alpha internal consistency results from the study proved the reliability of the questionnaire to be satisfactory with a value ranging from .84 for Maths-Tech scale to .92 for computer confidence. Although this scale appeared to be relevant for measuring computer and mathematics attitude for the current study with VET students the length of the questionnaire and assumptions about students’ experience of computer use in mathematics learning made it unsuitable for the cohort of TAFE students in this study.
Another Australian scale for measuring affective factors in mathematics learning with technology was developed by Galbraith and Haines (1998) and includes six subscales for factors including Mathematics Confidence, Computer Confidence, Mathematics Motivation, Computer Motivation, Mathematics Engagement and Computer Mathematics Interaction. Each Likert type subscale consists of 8 items with a total of 48 items for six scales. The items listed in the Computer Mathematics Interaction scale render is useful only when students have been exposed to computer based mathematics learning during a course and as such it is not useful to administer this scale as a pre-test. These scales provide a useful means for investigating students’ attitude to computer use and to mathematics particularly in the context of undergraduate studies.

In a study of 1st year undergraduate linear algebra and calculus course where MATLAB was integrated into their learning Cretchley and Galbraith (2003) used a mix of computer and mathematics attitude scales (Fogarty, Cretchley, Harman, Ellerton, & Konki, 2001; Galbraith & Haines, 1998) to investigate students’ perceived abilities and attitudes towards mathematics and computers both separately and interactively. Using data obtained from 196 students who took the pre-test and 82 students who took the full set of pre- and post-test and examinations Cretchley and Galbraith analysed the results to demonstrate associations between different affective and cognitive factors. The study found that mathematics attitudes (confidence and motivation) correlated strongly (p=0.65) with levels of achievement on a wide range of mathematics tasks. But computer attitude and mathematics attitude correlate weakly (p=.12). The study found only a weak correlation between computer confidence and motivation levels and performances generally on mathematics tasks in a technology rich learning environment. Mathematics tasks requiring use of technology were generally well done by the majority of the students – not only by those who were confident with and enjoyed using computers. The authors noted that computer confidence was a poor indicator of the likelihood of a mathematics student feeling empowered by the use of technology in learning mathematics. It appears from this study that harnessing students’ computer confidence for improving mathematics learning is a continuing challenge especially for those students who seem to show high computer confidence but low mathematics confidence.
Another commonly held belief amongst mathematics educators is that interest and enjoyment in mathematics learning would naturally lead to improvement in mathematics performance. But research shows that the relationship between interest and enjoyment in mathematics and mathematics performance is not clear-cut. Analysis of results from PISA study (Thomson, Cresswell, & Bortoli, 2004) shows only a weak relationship ($r=0.19$) between the interest and enjoyment in mathematics and mathematics literacy performance in Australian students. This weak relationship does not appear to be surprising because it was not significantly different from the OECD average and was similar to other English speaking countries. Interestingly, three of the highest performing countries - the Netherlands, Finland and Korea had means of the interest and enjoyment in mathematics index lower than the OECD average showing that students in these countries performed at a high level in mathematics but expressed less interest and enjoyment in mathematics than students in other OECD countries. However two other high performing countries Hong Kong and (Gadanidis, Gadanidis, & Schindler) had scores similar to OECD average. The PISA study used a four-item scale to measure interest and enjoyment in mathematics as an indicator of intrinsic motivation to study mathematics. The interest and enjoyment scores obtained in the PISA study illustrate the difficulty of associating interest and enjoyment measures with performance in mathematics tests with any consistency.

Although some researchers have challenged the validity of the assumption that positive attitude towards mathematics leads to better performance in learning mathematic because it may well be the case that students better performance in mathematics leads them to demonstrate a more positive towards mathematics (Galbraith, Pemberton, & Cretchley, 2001) there is a wide body of research that supports the view that positive attitude towards mathematics and performance correlate strongly. Crechley and Galbraith (2002) point out that the mathematics teachers’ goals of cognitive gains must be tempered by attention to affective outcomes. They assert that if a learning experience is unpleasant for the student, any gains in cognitive achievement and performance may be offset or diminished by attitudinal losses and this dislike or feeling of inadequacy may deter the student from studying further in the area. This critical balance between cognitive and affective goals is particularly important for contexts with adult learners in vocational education.
where an undue emphasis of obscure traditional mathematics content learning could easily ‘turn off’ a student who may not have pleasant experiences of learning mathematics from school years.

Affective factors such as attitudes and motivation play an important role in mathematics learning and McLeod (1992) in a review of research on affective factors points out that research on mathematics education can be strengthened by integration of affective issues in the studies of cognition and instruction. This research study incorporates a design experiment where an online learning environment was developed and used to assist the teacher in creating opportunities for making improvements in both affective and cognitive domains of learning. Considering the significance of affective outcomes for mathematics learning this study monitored students’ attitude towards mathematics by using pre- and post-test measures and implementing teaching strategies to foster positive attitude towards mathematics.

2.4. Technology and Mathematics Learning

Although the use of computers in teaching and learning of mathematics education research has been reported for a long time now it is only in the recent past that problem solving, simulation and micro-world based programs have emerged that try to engage learners in a context based problem solving situation and aim to teach mathematical concepts. The development of network based computing, the Internet and the world wide web offers new opportunities for the use of computers in mathematics teaching and learning and research into the range of ways and effectiveness of these new technologies is only beginning to emerge. In the following sections I will present a discussion on the theoretical perspectives relevant to the application of new learning technologies on the teaching and learning of mathematics with relevance to the vocational education and training context. In the first part I have discussed the work done by the Cognition and Technology Group at Vanderbilt (1992) in developing the theory of anchored instruction and the work of Collins, Brown and Newman (1989) in developing the model of cognitive apprenticeship as useful theoretical perspectives framing the design and development of my research project. The section on theoretical perspectives is followed by a discussion and
analysis of current perspectives on the role and impact of new learning technologies on mathematics learning. In the final part of this section a discussion on research literature related to the role of teacher in technology enriched teaching and learning of mathematics is presented to identify a relevant framework for analysing the teacher’s role in the current research.

2.4.1 Theoretical Framework

Based on theoretical foundations of constructivist learning and situated cognition, John Bransford (1990) guided the development of a model of learning known as anchored instruction at the Cognition and Technology Group at Vanderbilt (CTGV) where technology rich environments are used in learning mainly science and mathematics (CTGV, 1992). Anchored instruction ideas evolved in response to the inert knowledge notion associated with traditional approaches to instruction in education literature from the early twentieth century. Inert knowledge is knowledge that can usually be recalled when people are explicitly asked to do so but that is not used spontaneously in problem solving contexts even though it is relevant.

Anchored instruction uses technology-based innovations to situate learning in realistic problems, allowing students to experience the same professional dilemmas as faced by experts in a given field. Using learning environments where problems are structured to be factually authentic with real data and performances authentic with realistic tasks that might be faced by a novice being apprenticed to be an expert, anchored instruction shows similarities with case-based learning (CTGV, 1992). In anchored instruction stories or situations presented are meant to be “explored and discussed rather than simply read or watched” (CTGV, 1992, p. 249). It is similar to problem-based learning but does not expect students to do first-hand research into resources external to the learning environments. Instead anchored environments typically embed all of the information needed to solve the problem and make it easier for learners to interact in environments with limited time or limited resources.

Development of anchored instruction environments derives from the situated cognition framework (J. S. Brown, Collins, & Duguid, 1989) and relies on the use of authentic tasks in learning contexts. The authors noted that authentic activities are
commonly "ordinary practices of the culture" (p.34). In anchored instruction authenticity of tasks and activities can be analyzed from a number of perspectives. At one level it relates to the authenticity of objects and data being used in the learning context. At another level it involves the degree to which the task that students are asked to perform are authentic. At yet another level authenticity relates to the process of doing mathematics, that is how do the process of doing mathematics in class differs from how mathematics is done in real life contexts. Resnick and Klopfer (1989) pointed out that everyday settings are different from school settings in a number of ways. The school environment places more emphasis on individual work whereas in everyday work settings people have to perform tasks in collaboration with others. There is generally an emphasis on abstract reasoning in schools whereas in everyday settings contextualised reasoning is used more often. In addition, the school environment focuses more on ‘mental work’ whereas everyday work settings use tools to solve problems. Anchored instruction uses technology to bring the elements of contextualized everyday settings into classrooms by encouraging authentic tasks and collaborative work in solving problems that are located in real life contexts.

When a subject is taught in multiple contexts and includes examples that demonstrate wide application of what is being taught, people are more likely to abstract the relevant features of concepts and to develop a more flexible representation of knowledge (Gick and Holyak, 1983 as cited in Bransford, Brown, & Cocking, 1999, p. 50). But overly contextualised knowledge may not support effective transfer as shown by studies at Cognition and Technology Group at Vanderbilt (1992). For example in a study students learned mathematical concepts of distance-time-rate in the context of solving a complex case involving planning for a boat trip. The findings indicated that when students learned only in this context, they often failed to transfer flexibly to new situations. (CGTV 1997, as cited in Bransford, Brown, & Cocking, 1999, p. 50). The transfer of learning literature suggests that the most effective transfer may come from a balance of specific examples and general principles, not from either one alone (Bransford, Brown, & Cocking, 1999).

Although the anchored instruction model of learning presents real opportunities for authentic learning in classrooms and has been shown to promote higher order thinking skills, the requirements of standardised curriculum in educational settings pose a
challenge for educators willing to adopt this model. The technology-based macro-
environments proposed in anchored instruction are more suited to school aged
children. However, the concept of using technology to bring authentic tasks, real life
contexts, collaborative learning and problem solving are important elements of
anchored instruction and have helped in guiding the development of the online
learning environment for this study.

Another theory and model of learning that informed this research project is known as
cognitive apprenticeship (Collins, Brown, & Newman, 1989). Also based on
constructivist approaches to learning and situated cognition, the theory of cognitive
apprenticeship argues that people who have mastered certain cognitive skills often fail
to make them explicit when teaching to novices. It holds that it is important to bring
these “tacit processes into the open, where students can observe, enact, and practice
them with help from the teacher” (p.460). Cognitive apprenticeship methods require
externalisation of processes that are carried out internally by experts. According to
Collins, Brown and Holum (1991) cognitive apprenticeship methods of instruction
include:

- **Modelling**: it involves an expert’s performing a task so that the students can
  observe and build a conceptual model of the processes that are required to
  accomplish it.

- **Coaching**: it consists of observing students while they carry out a task and
  offering hints, scaffolding, feedback, modelling, reminders, and new tasks
  aimed at bringing their performance closer to expert performance.

- **Scaffolding**: it refers to the supports the teacher provides to help the student
  carry out the task. It may involve the teacher in executing parts of the task that
  the student cannot yet manage and gradual fading and removal of supports
  until students are on their own.

- **Articulation**: it includes any method of getting students to articulate their
  knowledge, reasoning, or problem-solving processes.
Reflection: it involves enabling students to compare their own problem solving processes with those of an expert or another student and ultimately, an internal cognitive model of expertise.

Exploration: it involves pushing students into a mode of problem solving on their own. Exploration is a natural culmination of the fading and supports. (Collins, Brown and Holum, 1991, p.14)

In this model while modelling, coaching and scaffolding are teacher led activities in making explicit the content and process, the articulation, reflection and exploration stages involve learners in actively refining and fine tuning their thinking and problem solving.

According to Collins, Brown and Newman (1989) cognitive apprenticeship involves the retooling of a traditional apprenticeship model for the teaching and learning of cognitive skills particularly aimed at teaching students the thinking and problem solving skills involved in school subjects like reading, writing and mathematics. Similar to anchored instruction articulated by Bransford (1990) cognitive apprenticeship also places a strong emphasis on the context of learning. Taking a lead from traditional apprenticeship practice where apprentices learn skills in the context of their application to realistic problems within a culture focused on and defined by expert practice, cognitive apprenticeship also encourages students to carry out tasks and solve problems in an environment that reflects the multiple uses to which their knowledge will be put in the future. The situatedness of learning in cognitive apprenticeship allows students to understand the purposes or uses of the knowledge they are learning, engages them in actively using knowledge rather than passively receiving it and offers them an opportunity to learn about the different conditions under which their knowledge can be applied (Collins, Brown, & Holum, 1991).

The proponents of the cognitive apprenticeship model of learning point out that computer technology enables creation of learning environments that can help in visualising and “realising the abstraction in practice” (Collins, 1988, p. 5). In addition, the computer also enables us to develop learning environments that “mimic situations
in the real world” that are otherwise impossible to realise in a classroom (Collins, 1988, p. 6).

The web based learning environment developed in this research project used these ideas to provide modelling and scaffolds with the help of computer simulated tasks for students. In addition the classroom practice adapted by the teacher also involved elements of cognitive apprenticeship by using modelling and coaching practices that allowed students to observe the details of problems solving and thinking processes associated with mathematical problem solving.

2.4.2 New Technologies in Mathematics Learning

In a review of technology applications in learning of mathematics in secondary schools Goos, Stillman and Vale (2007) identify a number of features of technology that afford learning opportunities in mathematics. They contend that new technological tools in mathematics provide opportunities to learn from observing patterns, instant feedback and making connections between multiple representations. In addition, the Internet offers opportunities to explore simulated or authentic data and offers the potential to extend learning by finding, sharing and communicating mathematics.

Citing numerous examples of spreadsheet and graphic calculator use Goos, Stillman and Vale (2007) point out that the “technology makes it possible for students to see connections between multiple representations of a concept” enabling the student to gain an understanding and insight into an abstract mathematical concept (p.78). The authors also highlight the usefulness of the Internet in teaching mathematics by pointing out that the Internet now makes available an enormous range of authentic data sets that allow learners to investigate mathematical problems using real life data sets. They also indicate that the Internet offers many useful sites where students can work with dynamic images of various kinds. Access to these dynamic images allows students to interact and manipulate these images in a virtual environment to develop an understanding of symmetry, transformation and other special concepts (p.80). Another key feature listed by Goos, Stillman and Vale (2007) is the use of the Internet as a networking tool which makes it possible for learners and educators to collaborate
and combine their efforts to solve mathematical problems and share their knowledge and experience with others.

In terms of effective teaching with the use of computer technology, Goos, Stillman and Vale (2007) assert “pedagogical content knowledge which enables teachers to create mathematical representations that connect students with subject matter is at the heart of teaching effectively with technology” (p. 100). They make it clear that “knowing how to use computers” is not enough to teach effectively with technology and teaching and learning of mathematics with technology entails much more than technical efficiency with computers.

Bransford, Brown and Cocking (1999) pointed out that new technologies, which include the Internet and web based applications, provide opportunities for creating learning environments that extend the possibilities of old and offer new possibilities but warns that technologies do not guarantee effective learning. They suggest that new technologies can be used in five ways:

- to create new opportunities for curriculum and instruction by bringing real-world problems into the classroom for students to explore and solve. For example connecting students with experts in the field, using interactive multimedia for learning
- to serve as scaffolds and tools to help students solve problems. Such as interactive simulators, calculators, virtual modelling
- to make it easier for teachers to give students feedback about their thinking and for students to revise their work. Network technologies for communication help make thinking visible.
- to connect classrooms to community both locally and globally.

Bransford, Brown and Cocking (1999) also noted that when teachers learn to use a new technology in their classrooms, “they model the learning process for students; at the same time they gain new insights on teaching by watching their students learn” (p. 195). Bransford’s ideas have continued to influence development and research of rich multimedia microworlds in teaching and learning in subject areas such as science and mathematics (Etheris & Tan, 2004; Shyu, 2000). One of the early examples of
Some fifteen years ago in a review of the role of technology in mathematics teaching and learning Kaput and Thompson (1994) visualised three aspects of electronic technologies that had the potential to enable a deep change in the experience of doing and learning mathematics. They identified interactivity, control and connectivity as three sources of power of new learning technologies. Interactivity offered by new learning technologies allowed learners to engage with and manipulate the learning environment in ways previously not possible in computing. The control available to designers of learning environments allowed them to “engineer constraints and supports, create agents to perform actions for the learner, make powerful resources immediately available to aid thinking or problem solving, provide intelligent feedback or context sensitive advice and control physical processes from the computer” (p.679). The connectivity power of new technologies makes it possible to link teachers to teachers, students to students, students to teachers and the world of education to the wider world of home and work. Indeed, studies of computer use in mathematics classrooms in recent years are reporting a significant decrease in the use of mathematical programs but a corresponding increase in the use of the Internet and spreadsheets (Thomas, 2006).

Ainley and Pratt (2006) pointed out that access to technological tools can support new approaches to the design of pedagogical tasks and at the same time provide new insights about the nature of mathematical understanding. They argue that technology offers us opportunities not only to teach the same curriculum in new ways but also to fundamentally challenge the current sequencing of some topics. Citing research from classroom use of contextualisation in mathematical tasks the authors claim that in many cases the supposedly real-world settings in classrooms are unable to achieve the desired outcome as the objectives of the task identified by the teacher may be quite
distinct from the one perceived by the learner. Cooper and Dunne (2000) as reported in Ainley, Pratt and Hansen (2006) indicate that in order to engage appropriately with the mathematical focus of a contextualised task, pupils have to understand complex but implicit rules about the extent to which they should attend to features of real-world settings. Ainley, Pratt and Hansen (2006) argue that providing tasks that may superficially offer authenticity by resembling out of school activities such as setting up a play shop in the corner of a classroom to encourage some mathematical learning are unable to provide the structure and constraints faced in a real shopping experience.

According to Ainley, Pratt and Hansen (2006, p. 29) purpose and utility are two important characteristics of learning environments. Purpose refers to the perceptions of the pupil and a purposeful task is defined as “one that has a meaningful outcome for the pupil, in terms of an actual or virtual product, or the solution of an engaging problem”. According to the authors, the purpose creates the necessity for the learner to use the ‘target knowledge’ (Brousseau’s term as cited in Ainley and Pratt, 2006) in order to complete the task. It may involve using existing knowledge in a particular way or constructing new meanings through working on tasks. Utility of mathematical ideas relates to the notion that mathematics learning encompasses not just the ability to carry out procedures, but the construction of meaning for the ways in which those mathematical ideas are useful. In the framework proposed by Ainley, Pratt and Hansen (2006) purpose and utility are closely connected because “appreciation of the utility of mathematical ideas can best be developed within purposeful tasks” (p. 30).

Using the notion of ‘planning paradox’ in the context of task design in computer based Microworlds and learning environments for mathematics learning, Ainley, Pratt and Hansen (2006, p. 24) contend that if teachers plan their lessons from tightly focused learning objectives, the tasks they set are likely to be ‘unrewarding for the pupils and mathematically impoverished’ but if teaching is planned around engaging tasks the pupils’ activity may be far richer but less focussed and difficult to assess. The authors offer three levels of resolution to the planning paradox. Firstly, the mathematics education in the school sector needs to rethink how to approach curriculum content to address content focus and motivation. Secondly, the contextualisation of mathematical activity needs to relate school and ‘real world’ experiences to stimulate a sense of purpose. Thirdly, attention needs to be paid to the
sorts of tools that teachers offer pupils whilst they work on the task. The affordances of these tools shapes the way pupils are able to pursue the teacher’s plans and understanding.

Although developed with relevance to school based teaching and learning of mathematics the notions of purpose and utility are very important for the teaching of mathematics in the context of vocational education. With a more direct connection of vocational education to industry it is not difficult to locate the utility of mathematical concepts developed but the issue of the ‘planning paradox’ remains a challenge for teachers of vocational mathematics when competencies and standards need to be carefully articulated and assessed.

Research by Trouche (2003), Lagrange (1999), Artigue (2002) and Hoyles, Noss and Kent (2004) provides another important perspective on the integration of digital technologies in mathematics learning. This perspective derives from work conducted in a range of quite different situations including activities with adults in workplaces and with students in classrooms and outlines the notions of instrumental genesis, orchestration and situated abstraction (Hoyles, Noss, & Kent, 2004).

The notion of instrumental genesis refers to the mutual transformation of learner and artefact in the course of constructing knowledge with technology. It contradicts the dominant opposition that exists between the technical and conceptual dimensions of mathematical activity where technology is conceptualised as a tool and technical artefact that releases the student from technical activity in order to focus on mathematical concepts (Artigue, 2002). Instrumental genesis is based on the notions of artefact and instrument where instrument is conceived as a psychological construct that a person operationalises in activity with an artefact in order to carry out some task (Hoyles, Noss, & Kent, 2004). According to Artigue (2002) instrumented activity involves two processes – instrumentalisation and instrumentation. In the process of instrumentalisation the subject shapes the artefact for specific uses and in a simultaneous process of instrumentation the subject is also shaped by actions with the artefacts. The “dialectic by which learner and artefact are mutually constituted in action is referred to as instrumental genesis” (Hoyles, Noss, & Kent, 2004, p. 313). The phenomenon of instrumental genesis offers a new insight into thinking about the
use of technology, including web-based learning resources, in teaching of mathematics and helps us conceive the technology not only as a master or servant but also as a partner or extension of self (Goos, Galbraith, Renshaw, & Geiger, 2003).

Another important element in the integration of technology in mathematics classrooms is the element of orchestration. The term as proposed by Trouche refers to the process of “external steering of student’s instrumental genesis” in order to enhance their learning of mathematics (cited in Hoyles, Noss, & Kent, 2004, p. 316). An example of orchestration reported by Trouche cited by Hoyles, Noss and Kent (2004) is how the architecture and organization of a mathematics classroom is presented. How the technology is configured to connect students, how students should sit in the classroom and which technologies should be switched on or off in order that individual instrumented actions can become the object of the collective as well as individual reflection and discussion. While recognising the importance of Trouche’s formulation of instrumented orchestration Hoyles, Noss and Kent (2004) placed a stronger emphasis on the role of interactions among learners and the role of technology in mediating these interactions. They highlight the importance of forming collaborative communities in classrooms that encourage sharing of different perspectives. The authors noted that computer based collaboration has been found to “encourage a shift in relationship between teacher and student and –under suitably managed circumstances with appropriate tools- enhanced task based interactions between students and between students and teachers”. They argued that technology being the medium plays an important role but it is students who ‘breathe life’ into the technologies and rebuild the mathematical structures for themselves by means of their actions on them (Hoyles, Noss, & Kent, 2004, p. 318). In this study it was important to pay attention to orchestration elements such as the design of the web-based learning environment as well as the organization of online activities to enable instrumented actions of students and follow it up with individual reflection and group discussion with the use of a discussion board.

Contrary to the conventional approach to learning where knowledge and skills are developed first and applied to problem solving subsequently, technology rich learning environments provide the possibility of learners working with concepts that they may not yet understand. Hoyles and Noss (cited in Ainley, Pratt, & Hansen, 2006, p. 29)
referred to this idea as ‘using before knowing…’ and claim that understanding emerges through activity. They argued that learning through use empowers students to learn mathematics in much the same way as learning in other natural contexts such as learning to read and write. This approach stands in contrast to the conventional approach in which the pupil rarely experiences using mathematics in meaningful ways (Ainley, Pratt, & Hansen, 2006). The use of online tools and calculators in exploring and problems solving activities with real life situations in this study was aimed at allowing students’ conceptual understanding to merge through their exploration and use of these tools.

With increasing prevalence of calculators, computers and networked technologies in mathematics learning another idea that has gained currency is the notion of affordances (J. Brown, Stillman, & Herbert, 2004; Watson, 2003). Gibson (1977) articulated affordances as relationships between objects and actors involved in interactive activity. Greeno (cited in J. Brown, Stillman, & Herbert, 2004) extends the notion of interactive activity and claims that affordance is “a property of whatever a person interacts with in the environment but this property must interact with a property of the person so as to support an activity” (p.121).

Brown, Stillman and Herbert (2004) pointed out that interactions between learners and technological devices necessarily involve both the ability of the learner and the affordance of the technology. They claim that it is these two elements that combine to determine the potential of the interactions in any given situation. Drijvers (2003) noted that the extent to which affordances of any technological tool can be realised in the classroom depends not only on the affordances of the technological tool but also on the way these affordances are managed by the teacher. Research has also shown that the teachers’ conception and representation of technological tools can easily transform affordances into constraints if the technological devices are used as a ‘black box’ rather than for exploration and sharing of ideas (Doerr & Zangor, 2000).

Yet, based on a design study of using computer modelling to teach algebra, Kennewell (2001) reported that constraints are not opposite of affordances. He claimed that in technology rich learning environments constraints can be constituted as complementary to affordances and within an educational setting learners can be
deliberately constrained in order to facilitate desired action. For example, the teacher could alter available affordances and constraints of a technological environment so that the gap between these and learner’s abilities allows intended learning to occur (Kennewell, 2001). Despite the fact that research literature on affordances of technology in mathematics learning is predominantly based on the application of graphical calculators and computer based software as learning environments, the principles involved are equally applicable to the design of web based learning environments to support mathematics learning in vocational education as well. For example, providing navigational constraints in web design so that students are able to use desired links from a website directly and not become distracted by irrelevant pieces of information allows a teacher to use the affordance of web based resources for novice users more effectively (Gerber & Shuell, 1998).

Addressing the issue of technology mediated learning in the context of secondary school mathematics learning Goos, Galbraith, Renshaw and Geiger (2003) contended that technology as a tool is integral to mathematical practice of teachers and students and electronic technologies such as computers and graphic calculators offer new opportunities for students to communicate and analyse their mathematical thinking. They point out that “technology can foster conjecturing, justification, and generalisation by enabling, fast, accurate computation, collection and analysis of data, and exploration of multiple representational forms (e.g., numerical, symbolic, graphical)” (p. 74).

Drawing on socio-cultural theories (Vygotsky, 1978) of learning Goos et al. (2003) asserted that technology is a cultural tool which mediates thinking and reasoning in the context of classroom. Technology as a cultural tool mediates learning in two ways. First, it serves to amplify existing classroom tools for learning by speeding up calculations or by verifying results obtained by pen and paper calculations. Second, it transforms learners’ thinking through technology mediated interactions and leads to re-organising of cognitive processes. Goos et. al. (2003) offer an example of this cognitive reorganisation by citing that “use of spreadsheet and graphing software can alter the traditional privileging of algebraic over graphical or numerical reasoning.” (p.75).
While acknowledging the instrumental genesis approach (Guin & Trouche, 1998) where students’ interactions with technological artefacts were assumed to have “transformed the material tools into an instrument of mathematical thought that reorganised their activity”, Goos et. al. (2003, p. 76) conceptualised the role of technology differently and place a greater emphasis on interactions that occur between teachers and students, amongst students and between people and technology. Based on a 3-year longitudinal study investigating the role of electronic technologies in supporting students’ exploration of mathematical ideas and in mediating their interactions with teachers and peers, Goos et. al. (2003) put forward a new analytical framework where they described classroom use of technology with the metaphors of *master, servant, partner or extension of self*. Based on varying degree and sophistication to which teachers and students interact with technology in their mathematical learning these differing perspectives are articulated by the authors as follows:

*Technology as master:* Technology can be labelled as a *master* when teachers’ and students’ knowledge and usage of technology is limited to a narrow range of operations relying on their technical competence (or lack thereof). In this formulation students are likely to develop dependence on technology because their lack of mathematical understanding prevents them from evaluating the accuracy of the output generated by these technological tools.

*Technology as servant:* Technology can be framed as *servant* when it is used only as a fast and reliable replacement for mental or pen and paper calculations. In this formulation the tasks of the classroom remain unchanged and technology is used as a supplementary tool to amplify cognitive processes. Students perceive technology as helpful in large and repetitive calculations because they can be carried out more quickly and efficiently and useful in reducing calculation errors and checking answers. Technology serves as a *servant* when it simply supports the preferred teaching methods and does not allow for creativity and exploration of ideas.

*Technology as a partner:* Technology serves as a *partner* when it is used creatively to “increase the power students exercise over their learning” (p.79). In this context the use of technology involves creative and exploratory tasks that facilitate understanding
and help in cognitive re-organisation based on exploration of different perspectives. Furthermore, technology is seen to act as a *partner* when it mediates mathematical discussion in the classroom.

*Technology as extension of self:* Technology is projected as an extension of self when users incorporate “technological expertise as a natural part of their mathematical and/or pedagogical repertoire.” (p.80). In this expression of technology students use and integrate a range of technological resources in building their mathematical models and arguments. The sophisticated application of computers and calculators in solving mathematical problems forms an “extension of the individual’s mathematical prowess” (p. 80).

Although Goos et.al. (2003) focussed on technology mediation in the context of secondary school mathematics and the use of graphing calculators and computers with mathematical software, their findings have theoretical and practical implications for mathematics teaching and learning for all sectors. They have shown that technological artefacts such as computers and calculators are not neutral objects and have the potential to reshape the interactions between teachers, students and the technology. Web based technologies offer additional power to classrooms to influence this interaction. Teachers have an important role to play in designing and supporting technology mediated learning experience for students where the role of technology is designed to shift from technology as master to technology as a partner and possibly even as an extension of self.

### 2.4.3 Teacher’s Role

Research has consistently shown that the teacher plays a central role in the educational process be it the behaviourist tradition of transmission of knowledge or a post-modern social constructivist model of experiential learning (Buzeika, 1996; Cashion & Palmieri, 2002; Hattie, 2003). Research reports from the vocational education sector in Australia also report that that the teacher plays a critical role in online learning and teaching styles that facilitate online learning in vocational education are strongly linked to teachers’ attitudes and their use of the medium (Brennan, 2003; Cashion & Palmieri, 2002). Stehlik, Simons, Kekham, Pearce and
Gronold (2003) researching the professional development needs of vocational education teachers for flexible and online learning reported that in order to deliver online programs well developed skills in writing, communicating, interpreting, conveying meaning and providing logical concise information are just as important as technological skills, such as the ability to use email, the Internet and PowerPoint applications. They clarified that the design and development of online courses however, does require a specific set of technical skills, as well as certain administrative and organisational skills. Brennan (2003) contended that the teacher’s role is critical in an online environment and technical, facilitation and management skills of teachers need to be combined in particular ways to suit the student, the content and the medium.

Robertson (2004) examined the practices of four TAFE teachers who used online technology in their classroom practice and found that these teachers adopted, applied and integrated online technology into their teaching practice in ways that supported their preferred teaching principles and did not have a negative impact on his or her teaching practice. In this study the selective adoption of features of online technology was demonstrated by the selective use of group email, bulletin board and computer marked assessments. One teacher used group email for communication with and between students as it aligned with her preferred teaching principle of developing a community of learners while another teacher’s preference for using computer marked tests supported her desire to implement reduction and repetition as a teaching strategy. The study provided a detailed look at how teacher’s self-declared preferred teaching principles are reflected in the selection and application of technology into their teaching practice.

Considering the role of teacher in computer facilitated learning in the post secondary context Berge (1995) proposed a useful model where the teacher’s role was categorized as pedagogical, social, managerial and technological. Bonk, Kirkley, Hara and Dennon (2001) applied this framework in online learning in higher education and identified practices that are likely to assist in the success of online learning including blended online learning where web based learning is mixed with traditional face to face teaching.
According to Berge’s framework the pedagogical role of the teacher is concerned with teaching and facilitating education processes for developing understanding of key concepts, ideas, and skills. In the managerial role teachers deal with organizational, administrative and procedural tasks and issues concerned with teaching in an online supported environment (Berge, 1995). In the social role teachers aim at promoting a friendly environment and a sense of community amongst learners and in the technical role they resolve technical issues related to course design and implementation (Bonk, Kirkley, Hara, & Dennen, 2001; Teles, Ashton, Roberts, & Tzoneva, 2001). This framework recognizes the range of issues teachers need to deal with when teaching in a technology supported learning environment and appears to be useful for a detailed analysis of the teacher’s role in a blended teaching and learning environment.

In a mathematics classroom, when integrating new learning technologies with traditional face to face learning, the issues of teachers’ technical skills, access to resources, organizational and structural support and their own instructional beliefs play an important role in effective design and implementation of technology supported learning (Ertmer, Addison, Lane, Ross, & Woods, 1999; Forgasz, 2006b; Handal, 2004).

According to Smith and Lovat (1995) the teacher’s instructional beliefs refer to the set of assumptions and ideas they hold with regards to the teaching and learning process. The curriculum and these set of assumptions are known to influence teachers’ practice and play a mediating role between the curriculum and instructional practice. Although Handal (2004) found obvious manifestations of constructivist and behaviourist orientations in teachers’ instructional beliefs and practices, other studies have shown that this dichotomy is not clear cut and the nature of the relationship between teachers’ mathematical beliefs and instructional behaviour is dialectical and highly complex (Buzeika, 1996; Clarke, 2005).

Research has also reported barriers to technology implementation in mathematics classrooms in terms of first and second order barriers (Ertmer, Addison, Lane, Ross, & Woods, 1999). The first order barriers are concerned with practical issues such as availability of hardware and software, availability of time to prepare instructional tasks and administrative support. The second order barriers relate to teachers’
instructional beliefs and attitudes concerning adoption and use of technology in the classroom. In this research the authors concluded that the second order barriers are crucial in determining the success of technology use because even when first order barriers are removed technology implementation can be severely affected by barriers such as teachers’ beliefs and attitudes.

Similarly Jackson and Anagnostropolou (2001) pointed out that improvements in learning through online approaches, when observed, are generally the product of reflective teachers who have conceptions that encourage them to develop effective teaching interventions regardless of technology rather than a feature of particular online pedagogy such as discussion groups or interactive exercises or hyper-linked resources. They assert that arguments attempting to claim that pedagogical improvements inherently follow from the use of online technologies are dangerously misleading. This argument has found widespread support in research literature (Biggs, 1999; Cashion & Palmieri, 2002) and especially in reference to mathematics learning Lynch (2006) pointed out that a lot of rhetoric surrounding the use of new technologies in schooling “overestimates the degree of agency that a technological artefact may have and fails to account for the agency of human actors, the effects of existing systems and institutions, and the complexity of interactions that influence how new technologies are used”(p.33).

Goos (2006) argued that simple notions of access and use are inadequate for developing an understanding of the role played by technology in mathematics teaching and learning. She claimed that teacher’s interpretation of access to technology is based on their beliefs about what is beneficial for students and feasible in the light of their own experience and expertise and institutional context.

Focusing on pedagogy and the nature of teacher’s professional learning Goos (2006) adapted Valsiner’s framework (1997) to analyse the interactions between teachers, students, technology and the teaching learning environment. Drawing on socio-cultural theories of learning, this framework places emphasis on learning as a product of interactions with other people and materials and tools situated in a particular learning environment. It extends the concept of Zone of Proximal Development (ZPD) originated by Vygotsky to include two more zones – Zone of Free Movement
(ZFM) and Zone of Promoted Action (ZPA). While Vygotsky referred ZPD to the gap between learner’s current capabilities and the potentially higher level performance that can be achieved with appropriate assistance, the ZFM and ZPA relate to the social setting and the goals and actions of participants.

Applying the notions of ZPD, ZFM and ZPA to technology enhanced learning of mathematics in the school sector Goos explained that the ZPD involves elements such as skill and experience in working with technology, pedagogical knowledge related to technology integration and general pedagogical beliefs; the ZFM applies to issues such as access to hardware, software, curriculum and assessment requirements and students motivation, behaviour and abilities; the ZPA deals with issues such as professional development and represents the efforts of a more experienced or knowledgeable person in the promotion and development of new skills. According to Goos (2006) “for learning to be possible the ZPA must be consistent with the individual’s potential (ZPD) and must promote actions that are feasible within a given ZFM”.

While Berge’s (1995) framework to describe teacher’s role in technology supported learning focuses on skills, knowledge and responsibilities of teachers and how technology implementation adds new dimensions to the role of teacher, Goos’ (2006) framework of analysing teacher’s role focuses more on analysing the interactions that occur within a school based learning context. In discussing the teacher’s role in technology supported mathematics learning I think it is important to explore not only how the technology adds new dimensions to a teacher’s role but also how the interactions within the learning context support or inhibit effective integration and use of technology. Both these frameworks are useful for analysing the teacher’s role in technology enriched mathematics learning and have guided this research in describing and analysing teacher’s practice.

2.5. Online Learning Environments in Mathematics
Advances in modern technology have enabled educational developers to create increasingly sophisticated and powerful tools for teaching and learning of mathematics. Computers and graphics calculators are now able to use software programs that have enormous potential to transform the content and nature of mathematics education (Forgasz, 2006b; Goos, Stillman, & Vale, 2007). During the past few years newer and more sophisticated mathematics software such as Derive, Maple, Mathematica and Geometer’s Sketchpad have become available in both secondary and post secondary education teaching of mathematics. There is no doubt that these technological tools and software can be effective and useful in teaching and learning of mathematics but their accessibility by students both in educational settings and in their own homes has been restricted by costs and stringent licensing issues (Loong, 2001; Wang, Kajler, Zhou, & Zou, 2003).

Mathematics educators and researchers have been interested in exploiting the potential of Internet based technologies for mathematics education for some time now and one can find an increasing number of mathematics related websites and interactive learning resources on the web. Software developers of mathematics learning have also realised the importance of providing content on the web and have extended the functionality of their stand alone software products to be web enabled. For example Mathematica and Cabri Geometry software are now available in WebMathematica and CabriWeb versions for a web based interface (Loong, 2001; Martinez, Barcena, & Rodriguez, 2005; Wang, Kajler, Zhou, & Zou, 2003). In the following sections I will discuss the design and development issues in producing mathematics learning content on the web and how current research and development is breaking new grounds in producing interactive content for web based mathematics learning. Later in the section I will also report on selected research from the school and higher education sectors where the web is used in teaching and learning of mathematics.

### 2.5.1 Designing Mathematics Learning on the Web

Loong (2001) in her study presented a topology of mathematics learning objects on the web identifying two main categories – the resources type and the communication type. The learning objects falling under the category of resource type are mainly those
objects that aim to explain and engage the learner in some form of mathematical content. Loong (2001) notes that interactivity of resources is the key discriminator in their educational usefulness and categorises these resources into three categories: Feedback systems, Exploratory Investigations and Games. She points out that non-interactive resources related to mathematics have been appearing from the early days of the Internet and serving varying purposes. These non-interactive resources have included materials like research articles, math history, lesson plans, archives of word problems, geometrical drawings, statistical tables and data, math jokes and various other static pieces of information related to mathematics. Communication technologies of the web offer another type of learning objects in mathematics. Math discussion forums, archives of questions and answers, and postings on various topics and levels in mathematics education use the features of the web to create a community of learners. The number of interactive, non interactive and communication based learning resources have been expanding both in quality and quantity since the publication of Loong’s taxonomy and according to recent estimates (Handal, Handal, & Herrington, 2006) currently there are more than 500 individual websites for teaching and learning mathematics. However, research in the area of publishing of mathematics content on the web and teaching of mathematics with Internet based technologies has been reported to be a difficult and more challenging task compared to other humanities based disciplines.

In a study of student attrition in mathematics e-learning courses in the United States Smith and Ferguson (2005) have shown that mathematics teachers experience considerably greater difficulties in communicating mathematics in a web based environment than teachers of other subjects since most online course management systems do not support mathematical notations and diagrams. In a study comparing attrition rates in online courses with face-to-face courses at the State University of New York the authors used a simple survey to obtain quantitative data on students enrolment and attrition from mathematics and non-mathematics related courses. From 138 responses obtained from online courses and 1246 responses obtained from face to face courses they noted that 32 online courses and 57 face-to-face courses were math related. In terms of attrition the study found that the attrition rate from online math courses (mean 0.31, SD 0.22) was significantly higher than non-math courses (mean 0.18, SD 0.14). However, in face-to-face courses attrition rate for math
related courses (mean 0.05, SD 0.06) was very similar to non-math courses (mean 0.05 SD 0.1). Although the students participating in this study were self-selecting, the findings point to a very obvious issue of difficulties in communicating mathematics in an online learning environment.

Smith and Ferguson (2005) reported that mathematics teachers in their study had to go through a three step process in order to post a mathematical notation on a web page – first they had to use a program like WebEq or MathType to generate a file with math notation, secondly, they had to save the file as an image file and finally they had to upload this image file on their web server to enable them to place this notation on the web page. They think that this process of importing mathematics notations into online documents made communication in online math courses extremely awkward. The issue of communicating mathematics in an online environment is a serious one and has affected uptake of online learning in a fully online mode. Smith and Ferguson (2005) also report that newer versions of online Learning Management Systems such as Blackboard and Firstclass have now integrated WebEQ plugins for writing mathematical notations and expressions but user familiarity and difficulties in learning new symbols and routines for online mathematics continue to pose problems for students.

Mavrikis and Macciocia (2005) report the development of a web based system to support learning of mathematics in the United Kingdom. Designed for students needing extra assistance in their mathematics learning in undergraduate programs this web based learning environment aimed at addressing the commonly perceived issue of growing deficiency in mathematical skills amongst students and the need for universities to take steps to address this problem.

Using Java applets and Java server based interactivity this web based learning environment offers HTML based text pages with embedded interactive objects on selected mathematics topics. These embedded objects are designed to provide visualisation of abstract concepts with animation and simulation and aim to create a dynamic learning environment for the learner. The WALLIS system also claims to support a feedback mechanism in order to avoid the need for a teacher to explain the task. The system uses interactive applets to communicate with a feedback frame to
provide task specific hints. The authors have developed this web-based system only as a prototype and piloted it with a small group of undergraduate students. The system appears to follow an instrumental pedagogy where the use of content and interactivity on the web is used to clarify and explain mathematical concepts to students. In particular those concepts that are fundamental in students successfully undertaking a science and engineering degree.

Mavrikis and Macciocia (2005) while acknowledging the potential of this system to support mathematics learning concede the need for more research on the development of feedback mechanism with learners to enable the development of a more detailed user model that tackles students’ actions and online help with the system more effectively. Lack of quantitative or qualitative data from this project also points to the need for a systematic study of student interactions with web based learning materials in mathematics and how these resources can be effectively integrated to support and enrich mathematics learning.

Advances in web programming with Java and XML tools and increased compatibility of Internet browsers to deal with specialised plugins to display mathematical objects have enabled developers to create web ready learning objects in mathematics. One such development from the field of vocational education in Australia is an innovative online support system designed to help in mathematics learning of vocational education students in Australia (Kavadias, 2003). Offering just in time learning assistance to students studying VET courses from the Building Industry Toolbox this online resource, branded as Your Online Learning Assistant (YOLA), delivers mathematics learning activities to students as they engage with online learning in their chosen vocational module.

A collection of learning objects to support this system contains activities from eleven topics including angles, area, conversions, fractions, length, measuring, multiplication and division, multiplying and dividing by ten, percentages, ratios and scale. These eleven learning objects are interactive and 'text-light' to support the needs of the young target audience enrolled in trade and vocational courses from the building industry.
In this resource, learning objects are presented to a learner only in the context of their vocational learning task. For example a student working with a building industry toolbox when learning about mixing mortar will be presented with a supplementary learning object on ratios by YOLA because the meta-tag for ratios is listed for the topic of mixing mortar. In this way the online support resource aims to offer just in time learning and practice in mathematics and avoids presenting a long list of topics for students to choose the desired activity.

As reported by Kavadias (2003) YOLA is not seen as a replacement of teacher input or intervention but it is rather seen as a first line of support to sustain an online learning experience and as a teaching tool. One key element of this online resource was to create a way to deliver just-in-time support. One of the key design features of this resource was that it used XML metadata instead of 'tagging' pages in existing resources. This design feature meant that adding the Learning Assistant (YOLA) to an existing resource required only the addition of one XML file rather than redeveloping pages of existing content making the online support content more portable. However YOLA and other metadata tools would not be able to match terms if one developer has a resource described by the term ‘ratio’ and a second resource is tagged by the term ‘ratios’ or ‘proportion’.

Despite considerable investment of resources in the development of the resource there is no published work available on the effectiveness of YOLA on a practical level and the extent to which students and teachers have actually adopted the system to benefit from the potential it offers.

Apart from individual efforts from members of the mathematics education community in designing and developing mathematics content for web-based education a number of institutions have also come to the forefront of interactive content development for use on the web. In this regard a significant contribution is made by a University of Utah through its National Library of Virtual Manipulatives project (National Library of Virtual Manipulatives Website, 2008). The National Library of Virtual Manipulatives (NLVM) offers a library of uniquely interactive web based virtual manipulatives or concept tutorials for mathematics instruction particularly suited to k-12 curriculum. These tutorials are similar to learning objects and use java
programming to offer interactivity for the learner. One of the primary aims of these interactive manipulatives is to help students visualise mathematical relationships and applications. This project has produced a large number of virtual manipulatives to teach mathematics topics including number operations, algebra, geometry, measurement, data analysis and probability. Furthermore, these Java based manipulatives are freely available to anyone from their website nlvm.usu.edu and do not require any special plugins for viewing in popular Internet browsers such as Internet Explorer or Mozilla.

Another institutional effort comes from the Learning Federation initiative by Australian and New Zealand governments (The Learning Federation, 2007). This initiative aims at producing a large range of interactive learning objects for school subjects including mathematics and numeracy. The Learning Federation has also developed a vast repository of more than 4000 learning objects and made it available free to Australian and New Zealand educational organisations. Managed by the Curriculum Corporation Australia this project has been a major initiative in online content development of a large number of learning objects that aim to involve teachers and learners in interactive learning activities. These learning objects are designed to be usable in educational settings as elements within larger units of work that may comprise other digital and non-digital materials. For example selected learning objects can be embedded in a website developed by a teacher.

A quasi experimental study by Freebody and Muspratt (2007) to evaluate the uses and effects of the learning objects in teaching of mathematics and science topics in 28 schools from Australia and New Zealand found that there was no reliable significant effect for learning objects use in mathematics. Both control and experiment groups achieved comparable performance across the board in almost every topic area covered during the study.

As noted by Freebody & Muspratt (2007, p. 62) the experimental findings and the site visits of their study suggests the “likely futility of attempting to make a simple case for or against the efficacy of learning objects, learning management systems, or their combination”. They point out that most significant educational problems are not soluble simply by the provision of better technologies. They point to the need for
more detailed, curriculum-specific and task-specific trials, research, and formal evaluation. The authors are keen to suggest that the research and development effort should now be directed to how learning objects can afford new interactional features and thereby new kinds of learning, and how those features can be put in place, in various ways, by teachers in various teaching contexts.

While learning objects developed by the Library of Virtual Manipulatives and the Learning Federation projects use latest technologies to create interactive multimedia content in mathematics that is usable by teachers and learners in a web-based learning environment, they are not customisable by individual teachers to suit their particular contexts. Once developed as a unit of web-based activity these learning objects cannot be edited or customised by teachers who wish to use them in their particular classrooms. However, this particular issue of teacher control in learning objects is being addressed by a project based at Kent State University.

The Institute for Computational Mathematics at the Kent State University in the United States is developing a Web-based Mathematics Education (WME) system to provide interactive mathematics content on the web that is easily portable and customisable by teachers (Wang, Kajler, Zhou, & Zou, 2003). Historically web pages using Hyper Text Markup Language (HTML) have not supported mathematical expressions for web publishing and as a result interactivity generated by CGI scripts, Javascript and similar programming languages has had problems in displaying mathematics calculations and expressions interactively on the web. The team working with WME framework has developed a mark-up language known as MeML. In the development of mathematical content for the web, the WME system uses open Internet technologies with MeML, HTML and MathML tags to create web sites that can produce mathematical expressions and figures and does not depend upon generation of image files for mathematical expressions using programs such as MathType (Zou & Wang, 2006).

The WME system supports mathematical formulas through MathML, interactive geometry objects through SVG (Scalable Vector Graphics), and a distributed mathematics assessment system called DMAS. The WME system focuses on interoperability and customisation of web-based mathematics content. Zou and Wang
(2006) claim that WME content can be readily combined to form different lessons and modules on the web. One important feature of these WME learning objects is that once they are developed they can be edited by classroom teachers to suit different contexts and in this way WME content for the web is claimed to be more advanced than other web based mathematics content currently available (Wang et al., 2008).

Currently WME system is being trialled in teaching mathematics in two secondary schools and one community and technical college in the United States and initial findings have been reported in conference presentations (Wang et al., 2008). These trials have been collecting data from teachers and students regarding the implementation of WME system in actual classroom situations to remove the bugs and improve the system. It can be argued that by allowing teachers of mathematics greater control over editing and customisation of content the WME system would help in extending the zone of free movement (Goos, 2006) of teachers and lead to the development of web based learning that encourages students to make sense of mathematical ideas by building rich connections to their existing knowledge and allows for the context in which tasks are developed to be meaningful for them (Wang et al., 2008).

Although the WME framework was not available during the designing and development of the online learning environment for my study, CGI and Java based interactive objects to support and enhance mathematical understanding of TAFE students were used. Our web content relied on images generated by MathType to display mathematical expressions that were incorporated with mostly static content for describing mathematical models and concepts. In addition the online learning environment for this study also made use of communication capabilities of the Internet to engage in writing and communicating mathematical ideas related to classroom activities. As recommended for further investigation by Freebody & Muspratt (2007) through the development and trial of this online learning environment in mathematics for TAFE students this study made an attempt at exploring how the interactional features of mathematics on the web can be put in place in a particular teaching context and how it affects students’ learning.
2.5.2 Using the Web in Mathematics Learning

Despite a significant growth in the development and availability of web based interactive resources for mathematics education in recent years relatively few studies are reported in literature that focus on the actual use of these web based resources in teaching and how mathematics teaching, learning and assessment is affected by the use of these web based resources. In more recent years papers from innovative teachers from both school and higher education sectors have begun to appear in conference proceedings but there is only very limited peer reviewed published work in the public domain. It appears that higher education sector has made more headway in terms of using the web for mathematics teaching and both undergraduate mathematics subjects and teacher education mathematics programs have been reported to apply web-based technologies in mathematics learning. Some papers from the primary school sector have also reported on the availability and use of web-based resources in teaching of primary school mathematics. This research study found only one project reported in literature where a web-based environment was designed to assist in teaching vocational mathematics (Kavadias, 2003). In the following section I will present a review of selected publications including conference papers and journal articles that relate to teaching of mathematics with the use of web-based learning resources.

Gunnarsson (2001) reported on the development and assessment of a graduate level statistics course taught in an online setting from a private mid-western Jesuit university in the U.S.A. The study involved two groups of forty-two students enrolled in a master in business administration course. One group was taught the subject content using a web-based format and the other group was taught with traditional face-to-face methods. The course covered principles and applications of descriptive and inferential statistics and aimed at making students familiar with basic techniques for understanding, organising, describing and computing research data. The study investigated students’ achievement in the course with respect to predictor variables such as their prior computer experience, their prior mathematics knowledge and experience, and their attitude towards the subject of statistics.
In Gunnarsson’s (2001) study the online version of the course used a virtual classroom using the Learning Edge option of Lotus Notes computer package. Using this virtual classroom, students accessed all course notes, assignments and PowerPoint presentations for the course. During the course students were given three collaborative group projects using real world data. Students used the online environment to access course content, engage in collaborative group discussions with other students and the teacher, post their assignments and view their course progress. Both the traditional face-to-face class and the online class using the virtual classroom were given the same collaborative projects, homework problems and examinations. In addition, the same teacher taught both classes. This study used questionnaires, individual interviews and an Attitude Towards Statistics test developed by Schau, Dauphinee & Del Vecchio (cited in Gunnarsson, 2001) to collect data and explore any relationship between achievement and predictor variables.

Findings from this study reveal that when the online class was compared to the traditional class the online class had more computer experience and a higher average positive attitude towards statistics than the traditional class. Students’ responses on the questionnaire also showed that the majority of them enjoyed online learning, loved the flexibility and did not feel isolated or detached. However there was a small group of students who despite agreeing with the benefits of online learning mentioned that they found it difficult to stay focussed and missed the interaction with other students and the instructor in the face-to-face format. However, in terms of achievement both groups produced comparable results and there was no clear advantage or disadvantage of using the virtual classroom in their final results.

Although this study was able to show clearly that students’ attitude towards statistics is positively affected by the use of an online learning format, it appears to have focussed only on the flexibility of access and collaborative communication aspects of the web-based environment. It did not appear to use any interactive online mathematics learning objects or tools to enhance students’ knowledge and skills of mathematics. In addition, the study also reported that some students did not favour a fully online course in mathematics and, while acknowledging the advantages of online learning, indicated their preference for face-to-face contact with their peers and the teacher. The study shows that blending face-to-face learning with an online learning
Engelbrecht and Harding (2004) reported on the use of web-based teaching of first year calculus courses in an undergraduate course from the University of Pretoria, South Africa. In this study the authors taught first and second semester calculus courses in a web-based mode where face-to-face contact was limited to one hour per week in a discussion mode. The course materials were provided via WebCT and a course book and students used email and a discussion forum to communicate during the course. The authors discuss the merits of combining online and paper-based assessment models and provide an analysis of results comparing students’ performance in both forms of testing. Student surveys are used to establish their preference for assessment.

The authors used a combination of online and paper based activities for both formative and summative assessments during this course. As part of formative assessment weekly online quizzes were done that provided instant feedback and results. In addition students were also required to do four hard copy assignments and a project activity for the course. The project required the use of Maple or Matlab software. The online quizzes consisted of Multiple Choice Questions (MCQ) and Constructed Response Questions (CRQ) but assignments consisted of CRQs only. During marking of tests while students were able to gain partial credits for their working out in CRQ type questions, no partial credit was possible with MCQ type questions where they were marked as either correct or incorrect. The final examination for the course also included both online and paper based components and equal credits were awarded to each component of the test.

While online assessment through quizzes focused on testing core mathematical skills and concept understanding using MCQs, the paper based assessment dealt with calculations or manipulations ranging over a number of steps. The results from the study comparing the performance in paper sections of tests with the online sections over a period of four semesters reported no “disturbing” difference between the online and paper sections (p.228). Although performance in online tests appeared to be slightly better than paper based tests the difference decreased gradually over four
semesters indicating that mastery of writing MCQs improves over time. The Pearson correlation coefficient between performances of individual students in the online and paper section of all tests changed from values of 0.10 and 0.13 in the first two semesters of testing to values of 0.58 and 0.35 in the last two semesters of testing indicating that the consistency between online and paper based results improved over time.

Survey results from this study also point to students overwhelming preference for combining online testing with paper based testing with more than 75% students preferring to allocate 50 to 70 percent credit to online assessment. Only 8.7% students said that no paper-based tests should be done. The main reasons for the preference of including online tests were that they provided immediate feedback and students could see if they needed to work more on the topic, offered flexibility in terms of when and where the test was taken and exposed them to use of computers in learning mathematics. The main reason given for preference for paper-based tests was the availability of partial credits.

The study clearly points towards students’ willingness to have a component of assessment in the online mode. In addition the study also shows that the quality of assessment does not deteriorate in the online mode although paper-based assessment in mathematics is able to assess skills such as formulation, exposition and sketching which are difficult to assess in the online mode.

In another study from the higher education sector Martinez, Barcena and Rodriguez (2005) designed a web based learning resource to help in teaching of geometry proof in a undergraduate course in Spain. Using CabriWeb feature of the Cabri-Geometre software the authors designed dynamic geometry objects that could be explored using a java compatible browser on the web. The Cabri-Géomètre software allows construction of geometrical figures that can be used for teaching and learning geometry. One of the key features of this dynamic construction is that the user continuously sees the figure redrawn in real time keeping all its initial properties. In this way Cabri lets the user conceptualize a figure not as a static drawing but as a set of objects linked by geometrical relationships (Martinez, Barcena, & Rodriguez, 2005).
The design of the learning environment used the java’s sketches to help students learn to develop problem-solving strategies and procedures and to apply them during problem solving. As an example, the study reports that classroom problems were chosen to highlight some properties of triangles and related tasks were designed to encourage students to search explanations with regard to the relationships that come across through the observation in the sketch. The tasks also assisted students in moving through exploration, prompting them to examine the evidence in the sketch and check the hint and the goals in the proof-problems. In this way the sequence of activities the authors used instructed students to discover unaided ideas for visual proofs that can be extended to pure mathematical proof. The authors report that the utilization of java applets has allowed them to use the time previously sent on constructing the figures in traditional methods for inquiry, in carrying out an investigation, discovering properties and geometric relations which promotes their geometric intuition.

Consistent with the notion of “using before knowing” (Noss & Hoyles, 1987, p.29) which assumes that understanding emerges through activity the study by Martinez, Barcena and Rodriguez (2005) encouraged students to discover unaided ideas for visual proofs and then extend this understanding to learning pure mathematical proofs.

In a study of using the Internet with mathematics teaching in teacher education courses in Brazil, Borba (2005) investigated the issue of how mathematical knowledge is modified by the use of technologies and the Internet chat. His study was based on the premise that knowledge is always constructed by collectives that involve humans and different technologies, and the interaction between technologies and humans has the potential to transform the knowledge that is produced (Noss and Hoyles, 1997, Borba 2005).

Borba (2005) noted that researching how collectives of humans-with-the Internet have constructed knowledge also transforms methodologies of research because data collection is more ‘natural’ in the Internet based educational environments when compared with face to face teaching. In the Internet environment recording of data
such as chat and email exchanges is automatic and issues of intrusiveness faced in
traditional face-to-face research methods are not of concern in Internet based
environments.

He reported use of online chat for two courses namely – ‘Trends in Mathematics
Education’ and ‘Teaching and Learning Geometry using software’. In each course
about 20 teachers who were dispersed geographically in remote areas of Brazil
participated in weekly chat sessions for a period of about four months. The courses
also used bulletin board and email messages to keep contact and support learning
sessions via chat. It is important to note that Borba’s research is based on an
experience of five years of using the chat sessions with these courses and it is not
surprising that the success of online chat sessions followed careful planning where
reading material or mathematical tasks were posted to students in advance and they
were expected to have completed them before the chat session. In addition, the
researcher and two nominated students from the class were given the role of raising
questions in the chat sessions to generate discussion. A third nominated teacher was
given the responsibility of summarising the chat session and posting it on the virtual
environment for the course.

Borba reported that Internet based chat sessions generated mathematics which would
be not possible in face-to-face teaching sessions. For example he notes that while
students using graphing calculators to generate conjectures for the problems related to
coefficients of parabola would not be writing their explanations in a face to face class,
in an online learning environment based on chat, writing is natural and doing
mathematics in this online environment has the potential to change the mathematics
produced because “writing in non-mathematical language becomes a part of doing
mathematics” (p.175).

The research by Borba showed that online chat can be successful when students are
engaged in a carefully planned and executed program that provides adequate
scaffolding for students’ participation in online chat sessions. Research by Smith and
Ferguson (2005) has however shown that in American distance education programs in
general there is much less success in teaching mathematics by use of Internet based
synchronous and asynchronous means. Their research showed that lack of support for
communicating mathematics symbols and expressions in the online medium plays a detrimental role. They argued that doing mathematics involves problem solving situations and discussion threads and chat sessions are not very successful with problem solving aspects of mathematics. Their findings have shown that there is a greater attrition of students from online mathematics courses in distance education compared with non-mathematics courses.

Although Borba did not mention the issue of difficulties of expressing mathematics in the online medium especially in chat sessions, it is clear that Borba’s focus of chat sessions for mathematics course was on discussing mathematics solutions rather than solving problems. It appears that teacher education and professional development courses where teachers are expected to discuss mathematics in addition to gaining skills and knowledge of solving mathematics problems, online chat has a legitimate role to play and could add a new dimension to mathematical knowledge as suggested by Borba.

In a study from a teacher education program from Canada where pre-service mathematics teachers were being encouraged to use Internet based java applets in their lessons Gadaniidis, Gadanidis and Schindler (2005) reported on the pedagogical thinking of teachers in using online applets in their lesson planning. The application of new media in this study was influenced by the thinking that mathematics teaching needs to find a pedagogical balance between listening to teacher exposition and practicing procedures with active investigation of mathematical relationships (Gadanidis, Gadanidis, & Schindler, 2003). The authors noted that new media not only enables us to express our ideas in new ways – they also affect the ideas we have, “We don’t always have ideas and then express them in the medium. We have ideas with the medium” (DiSessa cited in Gadaniidis, Gadanidis, & Schindler, 2003, p. 326).

The authors offered two sets of java applet programs to teach the topics of graphing linear equations and determining experimental and theoretical probability when rolling one die or two dice. The java applets offered simulated rolling of one or two dice with records of results and interactive graphing of a linear function of the form $y=mx + b$ and $y-y_1 = m(x-x_1)$. The pre-service students were shown the working of these applets and were given access to these applets to explore their function and use
in teaching of relevant mathematics topic. Students were then asked to make lesson plans for teaching the two mathematics topics and were asked to consider all available resources for teaching the topics. They were given the choice to use or not use the applets in their lesson plans. The lesson plans created by these pre-service students were analysed to explore their pedagogical thinking in hypothetical lesson planning situations.

Observations from the study suggested no consistent pattern in the use of applets in lesson planning during the study. Pedagogical beliefs of pre-service teachers reflected in their lesson plans showed that some believed in the need for teachers to first teach key concepts but others believed that students needed to first investigate mathematical relationships. In addition, the mathematics topic to be taught also influenced the nature of lesson plans, for example they found that a topic like linear functions was more likely to be treated in a traditional teacher centred way but a topic like probability was more likely to involve student investigation.

In one more study from the teacher education sector Li (2003) reported on the use of discussion forums in teaching a graduate course in mathematics for elementary schools in a Canadian university. The course involved regular face-to-face contact and participation in asynchronous computer conferencing. Ten student teachers participated in a semester long study.

Although the study did not provide data on the level of engagement by participants in the discussion forum it suggested that the forums provided a successful and novel experience to both the author and student teachers doing this course. The author noted that the use of discussion forum provides a more comfortable environment than face-to-face settings to discuss sensitive issues. The author also noted that “teachers (student teachers) talked frankly in threaded discussions about prickly and uncomfortable topics underlying education that I have never seen in their face-to-face engagements” p.76. Li (2003) also pointed out that threaded discussion also offered more choices by allowing students to choose the topic they wish to respond or create a new topic. In addition, Li (2003) noticed that discussions on the Internet forum became a reading and writing task as writing a comment took more time than saying it an oral conversation because most people liked to edit their messages before posting.
The author confirmed that more research was needed about understanding how students learn using the Internet.

Patahuddin and Dole (2006) reported on the use of Internet in an ethnographic study from a primary school in Queensland. In this study a teacher from a senior primary grade used the Internet to support and extend classroom based mathematics learning. The study explored the use of the Internet by the teacher in teaching of mathematics and what impact it had on students’ learning. The classroom where this study was conducted consisted of 26 students and six computers with only two of them connected to Internet. The teacher used the Internet connected computers strategically to engage students in mathematics research, to access learning objects and manipulatives, explore investigations, compare and communicate ideas with others, play maths games and solve maths puzzles.

The study noted three main reasons for the teacher’s use of the Internet in her class, namely “(1) to achieve her mathematics teaching objectives that mathematics is everywhere, (2) to facilitate student learning and (3) to have a good understanding and skill in using ICT” (p.404). The teacher reported that the use of the Internet impacted positively upon students’ learning. They were more engaged with learning demonstrated by their being more active in asking questions and discussing ideas. Students also collaborated with others and those with extra needs had the opportunity to extend or support their maths learning. The study also reported that initiatives to use the Internet in learning sometimes came from students and the availability of the Internet affected the teacher and made her role more of a facilitator of learning and through the Internet she was able to find the means to more easily engage students in learning and to cater to the various needs of different students. The study showed that the teacher’s skills and knowledge of computer and the Internet positively affected her success in integrating Internet based activities in her mathematics lessons. She was able to research and bookmark useful and relevant websites for easy access during the class time and had done her preparation for creating learning tasks that effectively guided the students to make use of the learning resources available on the Internet.

Ng and Hu (2006) in a study from a Singapore secondary school involving a group of gifted year 9 students, reported on teaching the topic of trigonometric curves using an
interactive web based simulation and following it with an online asynchronous discussion. During the study students worked in groups of three or four in experimenting with the web-based simulation to learn to sketch dilations and translations on trigonometric curves. The online discussion forum in the study was designed to help students decipher the concept behind each transformation using written dialogue and discussion.

The students first learned the concepts related to trigonometric curves during a 70-minute lesson where they completed four online activities on the topic with the help of the simulation Trigonometric Graph. In the following session they learned how to sketch transformations using Trigonometric Graph and took a 5-item paper-based quiz to test their knowledge. These two sessions were followed by four days of online discussions on the topic concluding with an oral examination at the end of a seven-day period.

Findings from the study indicated that the pass rate on the paper-based quiz was higher for this group compared to two previous classes who had received only traditional teacher centred instruction and did not use web based simulation activities. The authors reported that only 33% students from this experimental group managed to sketch horizontal translation in the paper-based quiz, which was drastically lower than what was found in the similar tasks in the online activity (79%). Similarly, only 41% could sketch combinations of transformation in the paper-based quiz whereas 79% of them were able to do a similar task in online activity (p.7).

The authors reported a positive impact of the use of the discussion forum because it allowed opportunities for students to reflect on their learning and also some students who otherwise would not take part in discussion were able to contribute via the discussion forum. The authors noted that although the results of the study suggested that students’ performance improved by using web-based simulations in learning trigonometric curves as shown by overall quiz and oral examination results, the anomaly presented by much higher success in online activity questions compared with paper-based quiz questions suggests that because concept based simulations are designed to explain a phenomenon they may help students understand why something
is happening and not how. They argue that the key to concept-based simulations is to ask students to explain “why” but not “how” questions (Ng & Hu, 2006, p. 9).

The study showed that use of web-based simulations and discussion is able to create opportunities for more constructivist learning in mathematics and reduce teachers’ dependence on expository methods. But more research is needed to establish how to assess and account for learning that is facilitated by new technological tools.

Khalid (2004) reported on a study in a technical college in Brunei where she taught mathematics to diploma in electrical engineering students using an innovative package. Although Khalid’s study did not make explicit use of web-based mathematics learning her package included activities where students had to use the Internet to explore and research particular concepts. Using lessons that emphasized an intuitive rather than rigorous instructional approach she adapted authentic real world and work contexts in her teaching. Her mathematics lessons on the topic of trigonometry for technical students included cooperative learning tasks and an integration of academic and technical subjects. The purpose of Khalid’s study (2004) was to look at the impact of changing the classroom environment in terms of instructional and pedagogical context on the attitude and mathematics achievement of students.

Participants of this study comprised two classes of students in their first year of National Diploma in Electrical and Electronics Engineering and National Diploma in Radio, Television and Electronics Engineering. These two classes were taught using a package on the topic of Trigonometry, designed and developed by the researcher. Using a mix of qualitative and quantitative measures cognitive and affective achievements of students were compared by pre-test and post-test for achievement; enjoyment and interest, and relevance and importance scales for attitude and a classroom environment survey. In addition classroom observations and student interviews were also conducted.

Findings from this study showed that one class had a statistically significant increase in pre-post test scores in cognitive achievement (t(11)= 5.59, p<.001) whereas the other class showed only a marginal increase (t(9) = 0.76, p<0.01). But, because the
post test was conducted after a number of weeks of instruction, it is naturally expected that students would show a better cognitive performance in post tests in comparison with their pre test performance. Results from pre-post attitude scores comparison indicated that while students mean score for interest and enjoyment scale increased, there was a significant drop in the scores for the relevance and importance scale. The author reported that not all students preferred the innovative teaching methods such as cooperative learning and problem solving and a small number of mature aged students with experience from the industry considered the new approach as a “waste of time”. The study reported that mature students who were comfortable with the traditional way of teaching seemed to resist changes. This study by Khalid (2004) highlights the problems associated with implementing instructional changes in a technical and vocational education setting and the difficulties in achieving empirical evidence for the effectiveness of intervention and innovation in mathematics teaching.

### 2.6. Conclusion

This chapter has presented a review of mathematics teaching and learning in the context of new learning technologies to illustrate how these new technologies offer an enormous potential to influence and transform mathematics learning for vocational education. The influence of new technologies in modern workplaces has increased the need of techno-mathematical literacies in workers and this literature review has shown that mathematics teaching in vocational education and training would potentially benefit from appropriate use of a technology enriched learning environment in mathematics teaching. In this research project the design and development of a web-based learning environment was influenced by this desire to enhance and transform mathematics learning for the vocational education students.

This literature review has also shown that although considerable research and development has taken place in recent years in terms of developing web-based interactive systems to teach mathematics in a range of contexts, the teaching of mathematics for vocational education has remained relatively un-researched (FitzSimons, 2002a). This review found only one more web-based learning environment (Kavadias, 2003) focusing on the teaching of mathematics in vocational
area. The review has also shown that second order factors play an important role in the success of technology use in classroom teaching and teacher’s skills, motivation and a close involvement in design and implementation of technology plays a crucial role in its effective use in the classroom. Using a design-based research methodology allowed this research to be closely linked to the practice of TAFE teachers of mathematics and helped in exploring and analyzing the role of teacher in technology enriched mathematics learning.

Developments in web-based learning technologies appear to have reduced the technological impediments in displaying and communicating mathematics on the web and research developments such as Web-based Mathematics Education systems offer greater control and interactivity to mathematics teachers for designing web-based learning environments. The review of classroom research studies from higher education and school sectors point to the need for studies that not only attempt to test the affect of technology use on achievement and attitude by quasi-experimental comparison but also to explore students’ interaction and use of these online learning environments in mathematics to determine factors that influence their participation and learning.

As a result the current study focuses not only on the design and development of a web-based learning environment for mathematics but also on exploring its use by mathematics teachers and students over a period of two research cycles as described in the next chapter on methodology.


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