**Frequency (signal) Filters** are electronic devices (analogue or digital) designed to suppress or attenuate particular frequency ranges within a signal.

The main types of filters are:
- Low pass – suppresses higher frequencies (eg anti-aliasing)
- High pass – suppresses lower frequencies (eg. Trend or drift removal)
- Band pass – suppresses all frequencies below and above a specific range (eg. To track a particular frequency within a broad-band signal)
- Notch – suppresses a particular band of frequencies (eg. To remove an undesirable frequency such as the mains power frequency.

Ideal filter characteristics: 

- **Low pass**
- **High pass**
- **Band pass**
- **Notch**
Actual filter characteristics:
Butterworth 2\textsuperscript{nd}, 4\textsuperscript{th} and 8\textsuperscript{th} order low-pass filters
(one order = 6dB/Oct or 20dB/Dec)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{butterworth_filters}
\end{figure}

\section*{Actual filter characteristics:}
Butterworth 2\textsuperscript{nd}, 4\textsuperscript{th} and 8\textsuperscript{th} order low-pass filters
phase lag = $\pi n/2$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{butterworth_phase}
\end{figure}
Actual filter characteristics:
Butterworth / Bessel, 8th order (48dB/Oct or 160dB/Dec)

![Amplitude vs Normalized Frequency Graph](image1)

Actual filter characteristics:
Butterworth / Bessel, 8th order (48dB/Oct or 160dB/Dec)

![Phase vs Normalized Frequency Graph](image2)
Actual filter characteristics:
Butterworth / Bessel, 8th order (48dB/Oct or 160dB/Dec)

Band-pass filters
Constant bandwidth: Every frequency bin has the same bandwidth

\[ f_c = \frac{f_l + f_u}{2} \]
Band-pass filters

Constant % bandwidth: The centre frequency is a geometrical mean of lower and upper cut-off frequency

\[ f_c = \sqrt{f_l f_u} \]
\[ \log(f_c) = 0.5 \left( \log(f_l) + \log(f_u) \right) \]

Octave filter: a specific type of constant % bandwidth filter: frequencies are internationally standardised and based on a reference of 1000 Hz

\[ f_u = 2f_l \]
\[ f_c = \sqrt{f_l f_u} = \sqrt{2} f_l \]
\[ f_l = \frac{f_c}{\sqrt{2}} \quad \text{and} \quad f_u = \sqrt{2} f_c \]
Band-pass filters

Decade filter: The upper and lower frequencies are separated by one decade:

\[ f_u = 10 f_l \]
\[ f_c = \sqrt{f_l f_u} = \sqrt{10} f_l \]
\[ f_l = \frac{f_c}{\sqrt{10}} \quad \text{and} \quad f_u = \sqrt{10} f_c \]